1 (c) (i) Factorise $y^2 - 2y - 48$

$y = \dfrac{2 \pm \sqrt{(-2)^2 - 4(-48)}}{2}$

$= \dfrac{2 \pm 14}{2}$

$y = 8$ or -6 (1). Hence, $(y+6)(y-8)$ (1)

$(y+6)(y-8)$

(2)

(ii) Hence, solve $y^2 - 2y - 48 = 0$

8, −6 (1)

(1)

(Total for Question 1 is 3 marks)

2 A bag contains X counters.

There are only red counters and blue counters in the bag.

There are 4 more blue counters than red counters in the bag.

Finty takes at random 2 counters from the bag.

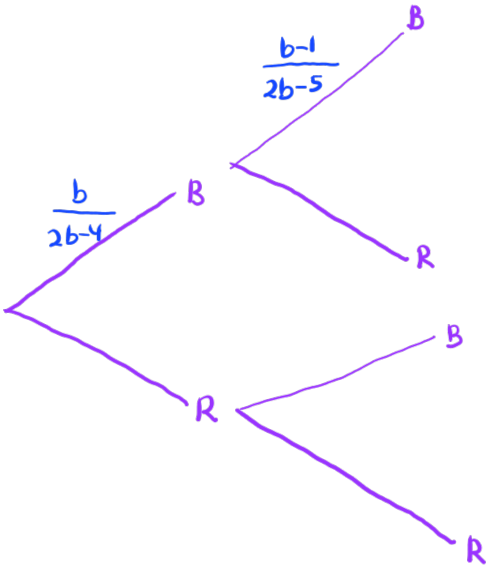
The probability that Finty takes 2 blue counters from the bag is $\frac{3}{8}$

Work out the value of X.
Show clear algebraic working.

$b + r = x$ $\quad$ $b = r + 4$

$b + b - 4 = x$ $\quad$ $b - 4 = r$

$2b - 4 = x$



$\left(\frac{b}{2b-4}\right)\left(\frac{b-1}{2b-5}\right) = \frac{3}{8}$ (1) (1)

$8b(b-1) = 3(2b-4)(2b-5)$

$8b^2 - 8b = 3(4b^2 - 10b - 8b + 20)$ (1)

$8b^2 - 8b = 12b^2 - 54b + 60$

$12b^2 - 8b^2 - 54b + 8b + 60 = 0$

$4b^2 - 46b + 60 = 0$ $\quad \div 2$

$2b^2 - 23b + 30 = 0$ (1)

$(2b - 3)(b - 16) = 0$

$b = \frac{3}{2}$ or $b = 10$

substitute b values into $(2b - 4 = x)$

$2\left(\frac{3}{2}\right) - 4 = x$ $\quad$ x has to be positive integers

$x = -1$ (not possible)

$2(10) - 4 = x$

$x = 16$ (1)

16

(Total for Question 2 is 5 marks)

3 Solve $x^2 - 5x - 36 = 0$
Show clear algebraic working.

$x^2 - 5x - 36 = 0$

1 $(x-9)(x+4) = 0$

$x-9=0$ or $x+4=0$

$x=9$ $\quad$ $x=-4$ 1

$-4, 9$ 1

(Total for Question 3 is 3 marks)

4 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$V = \dfrac{ds}{dt}$

$V = 3t^2 + 8t - 5$ (1)

$V = 3T^2 + 8T - 5$

$0 = 3T^2 + 8T - 5 - V$ (1)

$= \dfrac{-8 \pm \sqrt{(8)^2 - 4(3)(-5-V)}}{2(3)}$ (1)

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-8 \pm \sqrt{64 + 60 + 12V}}{6}$

$= \dfrac{-8 \pm \sqrt{124 + 12V}}{6}$ (1)

$= \dfrac{-8 \pm \sqrt{4(31 + 3V)}}{6}$

$= \dfrac{-8 \pm 2\sqrt{(31 + 3V)}}{6}$ (1)

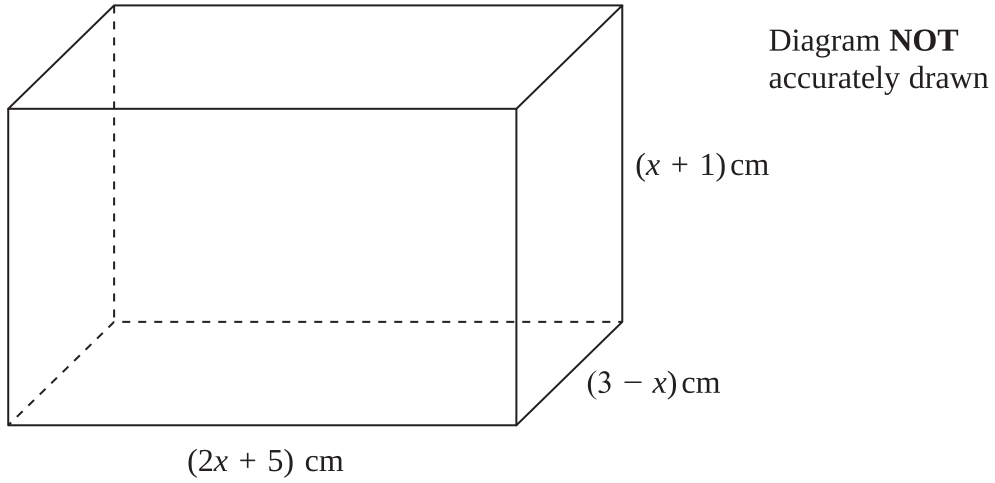
$= \dfrac{-4 \pm \sqrt{31 + 3V}}{3}$

$\therefore \dfrac{-4 + \sqrt{31 + 3V}}{3}$

$T = \dfrac{-4 + \sqrt{31 + 3V}}{3}$ (1)

(Total for Question 4 is 6 marks)

5



The diagram shows a cuboid of volume V cm^3

$V = 15 + 16x - x^2 - 2x^3$

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x.
Show your working clearly.
Give your answer correct to 3 significant figures.

Volume of cuboid is maximum when $\frac{dV}{dx} = 0$

$V = 15 + 16x - x^2 - 2x^3$

$\frac{dV}{dx} = 16 - 2x - 6x^2$ (1) (1)

$-6x^2 - 2x + 16 = 0$ (1)

By using quadratic equation :

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-6)(16)}}{2(-6)}$ (1)

$= \frac{2 \pm \sqrt{4 + 384}}{-12}$

$= \frac{2 \pm \sqrt{388}}{-12}$

$= \frac{2 + \sqrt{388}}{-12}$ or $\frac{2 - \sqrt{388}}{-12}$

$x = -1.81$ or $x = 1.47$

∴ x must be positive when cuboid is maximum. Hence,

$x = 1.47$ (1)

$x =$ 1.47

(5)

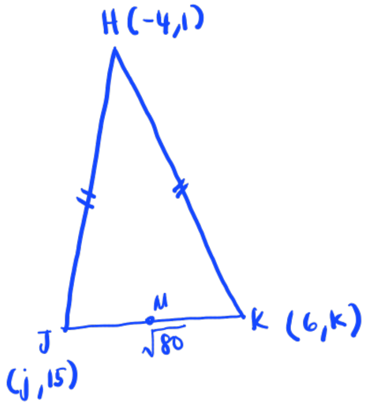
(Total for Question 5 is 5 marks)

6 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.



Given : gradient of HM = 2

gradient of JK $= \frac{-1}{2} = -\frac{1}{2}$ (1)

$$-\frac{1}{2} = \frac{(k-15)}{(6-j)}$$

$$-6+j = 2k-30$$

$$j = 2k-24 \quad \text{—} (1) \qquad (1)$$

Given : length of JK $= \sqrt{80}$

$$\sqrt{(6-j)^2+(k-15)^2} = \sqrt{80}$$

$$(6-j)^2+(k-15)^2 = 80 \quad (1)$$

$$j^2-12j+36+k^2-30k+225 = 80$$

$$j^2-12j+k^2-30k = -181 \quad \text{—} (2)$$

substitute (1) into (2) :

$$(2k-24)^2-12(2k-24)+k^2-30k = -181$$

$$4k^2-96k+576-24k+288+k^2-30k = -181$$

$$5k^2-150k+1045 = 0 \quad (1)$$

$$k = \frac{150 \pm \sqrt{(-150)^2-4(5)(1045)}}{2(5)} \quad (1)$$

$$= \frac{150 \pm \sqrt{1600}}{10}$$

$$= \frac{150 \pm 40}{10}$$

$k = 19$ or 11

substitute k values into (1)

$j = 2(19) - 24$ or $j = 2(11) - 24$

$= 14$ or $j = -2$

since $j < 0$,

$\therefore$ $j = -2$ and $k = 11$ (1)

$j =$ ......-2......

$k =$ ......11......

(Total for Question 6 is 6 marks)

7 (b) Solve $x^2 - 3x - 40 = 0$

Show clear algebraic working.

By using quadratic formula:

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-40)}}{2} \quad (1)$$

$$x = \frac{3 \pm \sqrt{169}}{2}$$

$$= \frac{3 \pm 13}{2} \quad (1)$$

$$x = \frac{3+13}{2} \text{ or } \frac{3-13}{2}$$

$$= 8 \text{ or } -5 \quad (1)$$

8, -5

(3)

(Total for Question 7 is 3 marks)

8 (i) Factorise $x^2 + 2x - 24$

$(x-4)(x+6)$

$(x-4)(x+6)$ 2

(2)

(ii) Hence solve $x^2 + 2x - 24 = 0$

$x = 4, -6$ 1

(1)

(Total for Question 8 is 3 marks)

9 Solve the simultaneous equations

$$y = 3 - 2x \quad - 1$$
$$x^2 + y^2 = 18 \quad - 2$$

Show clear algebraic working.

substitute 1 into 2 :

$x^2 + (3-2x)^2 = 18$ (1)

$x^2 + 9 - 12x + 4x^2 = 18$

$5x^2 - 12x + 9 - 18 = 0$

$5x^2 - 12x - 9 = 0$ (1)

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(5)(-9)}}{2(5)}$ (1)

$= \dfrac{12 \pm \sqrt{324}}{10}$

$= \dfrac{12 \pm 18}{10}$

$x = 3$ or $x = -0.6$

$y = -3$ or $y = 4.2$ (1)

$x = 3, y - 3$, $x = -0.6, y = 4.2$ (1)

(Total for Question 9 is 5 marks)

10 Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

Let black pens = B
red pens = R

$B = R + 3$

$B + R = N$

$(R+3) + R = N$

$2R + 3 = N$ (1)

$P(B) \times P(R) = \frac{9}{35}$

$\frac{R+3}{N} \times \frac{R}{N-1} = \frac{9}{35}$

$\frac{R+3}{2R+3} \times \frac{R}{2R+2} = \frac{9}{35}$

$\frac{R^2 + 3R}{4R^2 + 10R + 6} = \frac{9}{35}$ (1)

$35R^2 + 105R = 36R^2 + 90R + 54$ (1)

$R^2 - 15R + 54 = 0$ (1)

$R = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(54)}}{2}$

$= \frac{15 \pm \sqrt{9}}{2}$

$= \frac{15 \pm 3}{2}$

$R = 9$ or 6

$N = 2(9)+3$ or $2(6)+3$

$= 21$ or 15 1

21, 15

(Total for Question 10 is 5 marks)

11 Solve $x^2 - 21x + 20 = 0$

Show your working clearly.

By using quadratic formula:

$$x = \frac{21 \pm \sqrt{(-21)^2 - 4(1)(20)}}{2} \quad (1)$$

$$= \frac{21 \pm \sqrt{361}}{2}$$

$$= \frac{21 \pm 19}{2} \quad (1)$$

$$= \frac{21+19}{2} \text{ or } \frac{21-19}{2}$$

$$= \frac{40}{2} \text{ or } \frac{2}{2}$$

$x = 20$ or 1 (1)

20 , 1

(Total for Question 11 is 3 marks)

12 The functions f and g are defined as

$f(x) = x^2 + 6$
$g(x) = x - 10$

(b) Solve the equation $fg(x) = f(x)$
Show clear algebraic working.

$(x - 10)^2 + 6 = x^2 + 6$ 1

1 $x^2 - 20x + 100 + 6 = x^2 + 6$

$-20x + 106 = 6$

$100 = 20x$

$x = 5$ 1

5
....
(3)

(Total for Question 12 is 3 marks)

13 Solve the equation

$$\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$$

Show clear algebraic working.

$$\frac{5(x^2+2x) + 3(x+2)}{(x+2)(x^2+2x)} = 2 \quad (1)$$

$$5(x^2+2x) + 3(x+2) = 2(x+2)(x^2+2x) \quad (1)$$

$$5x^2+10x+3x+6 = 2(x^3+2x^2+2x^2+4x)$$

$$5x^2+13x+6 = 2(x^3+4x^2+4x)$$

$$5x^2+13x+6 = 2x^3+8x^2+8x$$

$$2x^3+8x^2-5x^2+8x-13x-6 = 0$$

$$2x^3+3x^2-5x-6 = 0 \quad (1)$$

$$(x+1)(2x-3)(x+2) = 0 \quad (1)$$

$$x = -1,\ 1.5,\ -2$$

Since $x+2 \neq 0$, x is equal to -1 and 1.5 (1)

ALTERNATIVE METHOD :

$$\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$$

$$\frac{5}{x+2} + \frac{3}{x(x+2x)} = 2$$

$$\frac{5x+3}{x^2+2x} = 2$$

$$5x+3 = 2(x^2+2x)$$

$$5x+3 = 2x^2+4x$$

$$2x^2-x-3 = 0$$

$$(2x-3)(x+1) = 0$$

$$x = -1 \text{ and } 1.5$$

-1 and 1.5

(Total for Question 13 is 5 marks)

14 (b) (i) Factorise $x^2 + 5x - 36$

$x^2 + 5x - 36$

$(x + 9)(x - 4)$ 2

$(x + 9)(x - 4)$
(2)

(ii) Hence, solve $x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$x + 9 = 0$ or $x - 4 = 0$

$x = -9$ $x = 4$

$4, -9$ 1
(1)

(Total for Question 14 is 3 marks)

15 A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds where $t \geqslant 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 5t^2 - 8t + 10$$

Find the displacement of P from O when P is instantaneously at rest.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

when P is at rest, $v = 0$

$\frac{ds}{dt} = 3t^2 + 10t - 8$ (1)

$0 = 3t^2 + 10t - 8$ (1)

$(3t-2)(t+4) = 0$ (1)

$t = \frac{2}{3}$ or -4 → t can only be positive, so $t = \frac{2}{3}$ is the only solution

$t = \frac{2}{3}$

$s = \left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) + 10$ (1)

$= \frac{194}{27}$ (1)

$\frac{194}{27}$ .... metres

(Total for Question 15 is 5 marks)

16 (b) (i) Factorise $x^2 + 8x - 9$

$x^2 + 8x - 9$

$(x - 1)(x + 9)$

2

$(x-1)(x+9)$
....
(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

$(x-1)(x+9)$

$x = 1$ or $x = -9$

$1, -9$ 1
....
(1)

(Total for Question 16 is 3 marks)

17 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

To get 5 counters still showing red face :

① First pick (R) + second pick (G)

② First pick (G) + second pick (R)

① $\frac{5}{x} \times \frac{(x-4)}{x} = \frac{5x-20}{x^2}$ (1)

② $\frac{(x-5)}{x} \times \frac{6}{x} = \frac{6x-30}{x^2}$

① + ② $= \frac{19}{32}$

$\frac{5x-20+6x-30}{x^2} = \frac{19}{32}$ (1)

$11x-50 = \frac{19}{32}(x^2)$

$32(11x-50) = 19x^2$

$19x^2 - 352x + 1600 = 0$ (1)

$(19x-200)(x-8) = 0$ (1)

$x = \frac{200}{19}$ or $x = 8$

$\therefore x = 8$ since $\frac{200}{19}$ is not a whole number

$x =$ ....8........ (1)

(Total for Question 17 is 5 marks)

18 (i) Factorise $x^2 + 5x - 24$

$(x-3)(x+8)$ 2

$(x-3)(x+8)$

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

$3, -8$ 1

(1)

(Total for Question 18 is 3 marks)

19 Here is a rectangle.

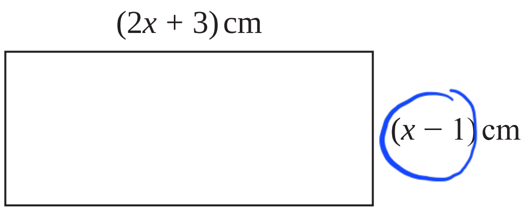


Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than 75 cm²

find the range of possible values of x

$(2x+3)(x-1) < 75$ 1

$2x^2 - 2x + 3x - 3 - 75 < 0$

$2x^2 + x - 78 < 0$ 1

$(x-6)(2x+13) < 0$ 1

$x = 6$, $x = -\frac{13}{2}$ is not a solution
1

$x > 1$ since length cannot be 0 or less.

Hence, $1 < x < 6$ 1

$1 < x < 6$

(Total for Question 19 is 5 marks)

20 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is a m/s^2

Find the value of a

$v = \frac{dx}{dt} = 12t^2 - 27 = 0$ (1)

$12t^2 = 27$

$t^2 = \frac{27 \div 3}{12 \div 3} = \frac{9}{4}$ (1)

$t = \pm\sqrt{\frac{9}{4}}$

$t = \pm \frac{3}{2}$ (1)

since $t \geqslant 0$, $t = \frac{3}{2}$

$a = \frac{dv}{dt} = 24t$ (1)

$a = 24\left(\frac{3}{2}\right)$

$= 36$ (1)

$a =$ 36

(Total for Question 20 is 5 marks)

21 (a) Factorise fully $18c^3d^2 - 21c^2$

$3(6c^3d^2 - 7c^2)$

$3c^2(6cd^2 - 7)$

2

$3c^2(6cd^2 - 7)$

(2)

(b) (i) Factorise $y^2 - 3y - 18$

$(y-6)(y+3)$

2

$(y-6)(y+3)$

(2)

(ii) Hence, solve $y^2 - 3y - 18 = 0$

$6, -3$ 1

(1)

(Total for Question 21 is 5 marks)

22 The radius of a right circular cylinder is x cm.

The height of the cylinder is $\left(\dfrac{800}{\pi x} - x\right)$ cm.

The volume of the cylinder is V cm^3

Find the maximum value of V
Give your answer correct to the nearest whole number.

$$V = \pi \times x^2 \times \left[\frac{800}{\pi x} - x\right] \quad 1$$

$$= 800x - \pi x^3$$

$$\frac{dV}{dx} = 800 - 3\pi x^2 \quad 1$$

$$800 - 3\pi x^2 = 0 \quad 1$$

$$800 = 3\pi x^2$$

$$x = \sqrt{\frac{800}{3\pi}} \quad 1$$

$$= 4914 \quad 1$$

4914

(Total for Question 22 is 5 marks)

23 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(b) Find the x coordinates of the points on **C** where the gradient is 4
Show clear algebraic working.

$\frac{dy}{dx} = 4 = 12x^2 + 2x - 20$ (1)

$12x^2 + 2x - 24 = 0$

$6x^2 + x - 12 = 0$ (1)

$(3x - 4)(2x + 3) = 0$ (1)

$x = \frac{4}{3}$ and $x = -\frac{3}{2}$ (1)

$\frac{4}{3}, -\frac{3}{2}$

(4)

(Total for Question 23 is 4 marks)

24 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(b) Solve gh$(x) = 1$

$gh(x) = \dfrac{11}{2(x^2+4)-5}$ 1

$1 = \dfrac{11}{2(x^2+4)-5}$

$2x^2 + 8 - 5 = 11$

$2x^2 = 8$

$x^2 = 4$ 1

$x = 2$ since $x \geq 0$

1

2

....

(3)

(Total for Question 24 is 3 marks)